Wir wählen eine Folge $(t_g)_{g\in N}$ mit

$$\sum_{g=1}^{\infty} t_g < \infty \quad \text{und} \quad \sum_{g=1}^{\infty} 2^{-g}\log t_g < \infty.$$

Dann ist fast überall

$$\log n_{g+1} \leqslant 2\log n_g + \log 3 - \log t_g$$

für $g \geqslant G(x)$ und weiters

$$\frac{\log n_{g+1}}{2^{g+1}} \leqslant \frac{1}{2}\sum_{k=G}^{g} \frac{\log 3 - \log t_k}{2^k} + \frac{\log n_G}{2^G}.$$

Also ist die Folge $2^{-g} n_g$ fast überall nach oben beschränkt.

Da $P\left(\frac{\log n_1}{2} > y \mid n_0\right) > 0$, folgt aus der Monotonie der Folge $2^{-g}\log n_g$ sofort

$$P\left(\lim_{g\to\infty} \frac{\log n_g}{2^g} > y \mid n_0\right) > 0$$

für jedes $y \geqslant 0$. Ähnlich wie W. Vervaat für Sylvestersche Reihen [4], kann man die Frage stellen, ob die Werte der Funktion $F: \boldsymbol{R} \to [1, \infty[$, fast überall definiert durch $F(x) = \lim_{g\to\infty} 2^{-g}\log n_g$ stetig verteilt sind.

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# Covering systems and generating functions

by

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This paper aims at giving some generalizations of basic properties known for exactly covering systems of arithmetical sequences to general systems of arithmetical sequences. After proving some general results concerning covering systems of sets of non-negative integers we introduce the concept of $(\mu, \mathfrak{m})$-covering systems of arithmetical sequences. The concept of $(\mu, \mathfrak{m})$-covering systems involves some notions concerning systems of arithmetical sequences investigated in the recent past, e.g. covering systems [1], exactly covering systems [1] and $\varepsilon$-covering systems [8].

**1. Preliminary results.** Let $\{f_n(z)\}_{n=0}^{\infty}$ be a sequence of complex functions defined on a region $D$ of the open complex plane $E$. Let the series $\sum_{n=0}^{\infty} f_n(z)$ be absolutely convergent for $z \in M$, where $M$ is a subset of $D$ having a point of accumulation in the region $D$. Since this series is absolutely convergent, all its subseries are also absolutely convergent for $z \in M$. Let $Z$ be the set of all non-negative integers. Let us suppose that to every non-empty subset $S$ of $Z$ appearing in our further considerations there exists a non-identically vanishing meromorphic function $f(S; z)$ defined on $D$ with

$$\sum_{n\in S} f_n(z) = f(S; z) \quad \text{for} \quad z \in M. \tag{1}$$

In case $S = \emptyset$ let us put $f(\emptyset; z) = 0$ for all $z \in D$. Since the set $M$ has a point of accumulation in $D$ and $f(S; z)$ with $S \neq \emptyset$ is not vanishing on the entire $D$, then $f(S; z)$ with $S \neq \emptyset$ is also not vanishing on the entire $M$.

Let us modify Definition 2 in § 1.1 of [6] in the following way. Let $S_1, S_2, \ldots, S_k$ $(k \geqslant 1)$ be subsets of the set $Z$ and $\mu$ a function defined on the system $\{S_1, \ldots, S_k\}$ with values in the set $\{-1, 1\}$. Let us put

$$D_m = D_m(S_1, \ldots, S_k) = \left\{n \in Z: \sum_{t=1}^{k} \mu_t \chi_t(n) = m\right\}$$

for every integer $m$, where $\chi_t$ is the characteristic function of the set $S_t$ and $\mu_t = \mu(S_t)$ for $t = 1, 2, \ldots, k$.

Thus the set $D_m$ consists of such integers which belong "in fact" to $m$ sets of the system $\{S_1, \ldots, S_k\}$ and therefore $D_m \cap D_n = \emptyset$ whenever $m \neq n$. We immediately get $D_m = \emptyset$ for $m$ with $|m| > k$. In case $\mu_t = 1$ for all $t$ we have for instance $D_0 = Z - \bigcup_{t=1}^{k} S_t$, $D_k = \bigcap_{t=1}^{k} S_t$ and if the sets $S_1, \ldots, S_k$ are mutually disjoint then $D_1 = \bigcup_{t=1}^{k} S_t$.

In what follows every system $\{S_1, \ldots, S_k\}$ of subsets of $Z$ will be supposed to be given together with function $\mu$ mentioned above. Moreover we restrict our attention only to systems $\{S_1, \ldots, S_k\}$ with

$$S_i = S_j \quad \text{implies} \quad \mu_i = \mu_j$$

for $i, j = 1, \ldots, k$. In the opposite case we can eliminate the sets not satisfying this condition without changing any of $D_m$'s.

LEMMA 1. *Let $\{S_1, \ldots, S_k\}$ be a system of sets of non-negative integers. Then*

$$\sum_{t=1}^{k} \mu_t f(S_t; z) = \sum_{m=-k}^{k} m f(D_m; z)$$

*in the sense of equality of meromorphic functions on the region $D$.*

Proof. Let $n \in Z$ and $\mathfrak{m}(n) = \sum_{t=1}^{k} \mu_t \chi_t(n)$. Then the series

$$\sum_{n \in Z} \mathfrak{m}(n) f_n(z), \qquad z \in M$$

is also absolutely convergent for the sequence $\{\mathfrak{m}(n)\}_{n=0}^{\infty}$ is bounded. We get the right-hand side of the required equality by rearrangement of this series according to the same $\mathfrak{m}(n)$'s and the left-hand side by its gradually rearrangement according to indices which are elements of the sets $S_1, \ldots, S_k$. Thus the required equation holds on $M$. But then it also holds on the entire region $D$, because $M$ has a point of accumulation in $D$.

The converse of this lemma will also be important for our further considerations, but it does not hold in general. In order to provide this converse we must add some conditions concerning the functions $f_n(z)$.

LEMMA 2. *Let $S_t, T_t, T_{-t}$ be subsets of $Z$ for $t = 1, \ldots, k$ and let the sets $T_{-k}, \ldots, T_k$ be mutually disjoint. Let the functions $f_n(z)$ satisfy the following condition*

(2) *if* $\sum_{n \in Z} a_n f_n(z) = 0$ *for all* $z \in M$, *then* $a_n = 0$ *for all* $n \in Z$.



*Then the equation*

$$\sum_{t=1}^{k} \mu_t f(S_t; z) = \sum_{t=-k}^{k} t f(T_t; z)$$

*implies*

$$T_t = D_t(S_1, \ldots, S_k) \quad \text{for} \quad t = -k, \ldots, k.$$

Proof. Owing to the previous lemma we have

$$\sum_{t=1}^{k} \mu_t f(S_t; z) = \sum_{t=-k}^{k} t f(D_t; z)$$

and hence

$$\sum_{t=-k}^{k} t f(D_t; z) = \sum_{t=-k}^{k} t f(T_t; z).$$

Let us remark that the sets $D_t$ $(t = 1, \ldots, k)$ are mutually disjoint. Then we get the statement of our lemma comparing the coefficients of both sides after expanding the functions $f(D_t; z)$, $f(T_t; z)$ into series (1) on $M$.

**2. Main results I.** Let $\mathfrak{m}(n)$ be a function defined on the set of all integers. Let the function $\mu$ be defined on the system of residue classes

(3) $$a_t \ (\mathrm{mod}\ n_t), \qquad t = 1, \ldots, k.$$

Then the system (3) is called to be $(\mu, \mathfrak{m})$-covering if

(4) $$\sum_{t=1}^{k} \mu_t \chi_t(n) = \mathfrak{m}(n)$$

for every integer $n$, where $\chi_t$ is the characteristic function of the residue class $a_t \ (\mathrm{mod}\ n_t)$ for $t = 1, \ldots, k$.

For instance, we obtain the concept of exactly covering system of arithmetical sequences if the functions $\mu$ and $\mathfrak{m}$ reduce to constants $\mu_i = 1$, $\mathfrak{m}(n) = 1$, [1]–[5]. The $(\mu, 1)$-covering system is just the $\varepsilon$-covering system of [8]. The well-known concept of covering system of congruences is obtained in case $\mathfrak{m}(n) > 0$, $\mu_t = 1$ for all $n$ and $t$.

It can be easily verified that the function $\mathfrak{m}(n)$ must be periodic. The period of this function will be denoted by $n_0$. If (3) is $(\mu, \mathfrak{m})$-covering then $n_0$ is a divisor of the l.c.m. $[n_1, \ldots, n_k]$ because of

$$\mathfrak{m}(n + [n_1, \ldots, n_k]) = \mathfrak{m}(n)$$

for every integer $n$.

The system (3) is $(\mu, \mathfrak{m})$-covering if and only if (4) holds for every non-negative integer $n$ (even if and only if (4) holds for every integer $n$ of any interval of the length $[n_1, \ldots, n_k]$). For this reason we may restrict our attention only to non-negative elements of the classes of (3) and so to use our preliminary results.

THEOREM 1. *Let $f(z)$ be a meromorphic function defined on a region $D$ of the open complex plane. Let $M^* = \{z \in D\colon 0 < |f(z)| < 1\}$. Let $M$ be a subset of $M^*$ having a point of accumulation in the region $D$ and let $f(z)$ be one-to-one on $M$. Then the following statements are equivalent:*

$A_1$. *The system* (3) *is* $(\mu, \mathfrak{m})$*-covering.*

$B_1$. $$\sum_{t=1}^{k} \frac{\mu_t f^{a_t}(z)}{1-f^{n_t}(z)} = \sum_{t=0}^{n_0-1} \frac{\mathfrak{m}(t) f^t(z)}{1-f^{n_0}(z)}$$

*in the sense of equality of meromorphic functions on $D$.*

Proof. This theorem follows from Lemmas 1 and 2 but we need to show that the sequence

$$1,\ f(z),\ f^2(z),\ f^3(z),\ \ldots \tag{5}$$

satisfies condition (2) on $M$.

In virtue of the uniqueness of the expansion of a function in the Laurent series ([7], Chapter III) we get

(6) if $\sum_{n \in Z} a_n z^n = 0$ in a circular neighbourhood of the point 0, then $a_n = 0$ for all $n \in Z$.

The set $M$ has a point of accumulation in the region $D$. Further, $f(z)$ is meromorphic on $D$ and it is everywhere defined and one-to-one on $M$ and therefore $f(M)$ has a point of accumulation in $f(D)$. Let $C(0)$ be a circular neighbourhood of the point 0 containing $f(M)$ and at least one of the points of accumulation of the set $f(M)$ in $f(D)$. If (5) does not satisfy (2) then we get a contradiction with (6) for this $C(0)$.

Let us introduce the symbol $\delta_{s,t}$ in the following manner

$$\delta_{s,t} = \begin{cases} 1 & \text{if} \quad t \mid sn_0, \\ 0 & \text{if} \quad t \nmid sn_0, \end{cases}$$

where $n_0$ is the period of the function $\mathfrak{m}$ and $|$ is the sign of divisibility.

THEOREM 2. *The following statements are equivalent:*

$A_1$. *The system* (3) *is* $(\mu, \mathfrak{m})$*-covering.*

$C_1$. $$\sum_{\substack{t=1 \\ n_j \mid sn_t}}^{k} \frac{\mu_t}{n_t} \exp\left(\frac{2\pi i}{n_j} s a_t\right) = \frac{\delta_{s,n_j}}{n_0} \sum_{t=0}^{n_0-1} \mathfrak{m}(t) \exp\left(\frac{2\pi i}{n_j} st\right)$$

*for $s = 1, \ldots, n_j$ and $j = 0, 1, \ldots, k$.*

$D_1$. $$\sum_{t=1}^{k} \mu_t n_t^{r-1} \cdot B_r\left(\frac{a_t}{n_t}\right) = \sum_{t=0}^{n_0-1} \mathfrak{m}(t) n_0^{r-1} \cdot B_r\left(\frac{t}{n_0}\right)$$

*for every $r \in Z$, where $B_r(x)$ is the $r$-th Bernoulli polynomial.*



Proof. Owing to Lemma 1 the statement $A_1$ is equivalent to the following one

$B_1^*$. $$0 = \sum_{t=1}^{k} \frac{\mu_t z^{a_t}}{1-z^{n_t}} - \sum_{t=0}^{n_0-1} \frac{\mathfrak{m}(t) z^t}{1-z_{n_0}}$$

in the sense of equality of meromorphic functions on $E$.

Let us denote the right-hand side of $B_1^*$ by $H(z)$. Evidently $\lim_{z\to\infty} |H(z)| = 0$. But then the equivalence between $B_1^*$ and $C_1$ follows from the Theorem of Liouville and the rule

$$\operatorname{res}_a \frac{F(z)}{G(z)} = \frac{F(a)}{G'(a)}$$

provided that $F(z)$, $G(z)$ are holomorphic and $G(z)$ has only single roots.

The equivalence of $A_1$ and $D_1$ follows from Theorem 1 for $f(z) = \exp z$, $D = E$ and from the expansion

$$\frac{z \exp(xz)}{\exp(z)-1} = \sum_{n \in Z} B_n(x) \frac{z^n}{n!}, \qquad |z| < 2\pi,$$

using the basic properties of absolutely convergent series and uniqueness theorem for meromorphic functions.

The part $C_1$ was proved for exactly covering systems in [5], $D_1$ for exactly covering systems in [3] and for $\varepsilon$-covering systems in [8].

**3. Main results II.** In this section we prove results analogous to those of the previous section taking the parity of moduli $n_1, \ldots, n_k$ of classes in (3) into account. Therefore we shall suppose (3) to be ordered in such a way that

(7) $n_1, n_2, \ldots, n_q$ are all even and $n_{q+1}, n_{q+2}, \ldots, n_k$ are all odd moduli of residue classes in (3).

THEOREM 3. *Let the assumptions of Theorem 1 be satisfied for the function $f(z)$. Then the following statements are equivalent:*

$A_2$. *The system* (3) *is* $(\mu, \mathfrak{m})$*-covering with* (7) *being satisfied.*

$B_2$. $$\sum_{t=1}^{q} \frac{(-1)^{a_t} \mu_t f^{a_t}(z)}{1-f^{n_t}(z)} + \sum_{t=q+1}^{k} \frac{(-1)^{a_t} \mu_t f^{a_t}(z)}{1+f^{n_t}(z)} = \sum_{t=0}^{n_0-1} \frac{(-1)^t \mathfrak{m}(t) f^t(z)}{1-(-1)^{n_0} f^{n_0}(z)}$$

*in the sense of equality of meromorphic functions on $D$.*

The proof is analogous to that of Theorem 1 using the sequence

$$1,\ -f(z),\ f^2(z),\ -f^3(z),\ \ldots$$

THEOREM 4. *The following statements are equivalent:*

$A_2$. *The system* (3) *is* $(\mu, \mathfrak{m})$*-covering with* (7) *being satisfied.*

$C_2$. $$\sum_{\substack{t=1\\ n_j|sn_t}}^{q} \frac{(-1)^{a_t}\mu_t}{n_t}\exp\left(\frac{2\pi i}{n_j}sa_t\right) = \frac{\delta_{1,2}\cdot\delta_{s,n_j}}{n_0}\sum_{t=0}^{n_0-1}(-1)^t\mathfrak{m}(t)\exp\left(\frac{2\pi i}{n_j}st\right)$$

*for* $j = 0, 1, \ldots, q$ *and*

$$\sum_{\substack{t=1\\ n_j|(2s+1)n_t}}^{k} \frac{(-1)^{a_t}\mu_t}{n_t}\exp\left(\frac{(2s+1)\pi i}{n_j}a_t\right) = \frac{\delta_{2s+1,n_j}}{n_0}\sum_{t=0}^{n_0-1}(-1)^t\mathfrak{m}(t)\exp\left(\frac{(2s+1)\pi i}{n_j}t\right)$$

*for* $j = \delta_{1,2}(q+1),\ q+1,\ q+2, \ldots, k$ *and* $s = 1, 2, \ldots, n_j$.

$D_2$. $$\sum_{t=1}^{q}\frac{2\mu_t}{n+1}(-1)^{a_t+1}n_t^r\cdot B_{r+1}\left(\frac{a_t}{n_t}\right) + \sum_{t=q+1}^{k}(-1)^{a_t}\mu_t n_t^r\cdot E_r\left(\frac{a_t}{n_t}\right) = n_0^r\sum_{t=0}^{n_0-1}\left[(1-\delta_{1,2})\cdot(-1)^t\mathfrak{m}(t)\cdot E_r\left(\frac{t}{n_0}\right) - 2\delta_{1,2}\frac{(-1)^t}{n+1}\mathfrak{m}(t)\cdot B_{r+1}\left(\frac{t}{n_0}\right)\right]$$

*for every* $r \in Z$, *where* $B_r(x)$, *resp.* $E_r(x)$ *is the* $r$*-th Bernoulli, resp. Euler polynomial.*

The proof is based on the ideas of the previous one and the expansion

$$\frac{2\cdot\exp(xz)}{\exp(z)+1} = \sum_{r\in Z} E_r(x)\frac{z^r}{r!}, \qquad |z| < \pi.$$

**4. Corollaries.** In this section we give some corollaries of the previous results. Some of them generalize known results.



COROLLARY 1. *Let* (3) *be a* $(\mu, \mathfrak{m})$*-covering system. Then*

1. $$\sum_{t=1}^{k}\frac{\mu_t}{n_t} = \frac{1}{n_0}\sum_{t=0}^{n_0-1}\mathfrak{m}(t),$$

2. $$\sum_{t=1}^{k}\mu_t\frac{a_t}{n_t} = \sum_{t=1}^{k}\frac{\mu_t}{2} + \sum_{t=0}^{n_0-1}\mathfrak{m}(t)\left(\frac{t}{n_0} - \frac{1}{2}\right),$$

3. $$\sum_{t=1}^{q}\frac{(-1)^{a_t}\mu_t}{n_t} = \frac{\delta_{1,2}}{n_0}\sum_{t=0}^{n_0-1}(-1)^t\mathfrak{m}(t),$$

4. $$\sum_{t=1}^{q}2(-1)^{a_t+1}\cdot\mu_t\left(\frac{a_t}{n_t} - \frac{1}{2}\right) + \sum_{t=q+1}^{k}(-1)^{a_t}\mu_t = \sum_{t=0}^{n_0-1}(-1)^t\mathfrak{m}(t)\cdot\left(1 - 2\delta_{1,2}\frac{t}{n_0}\right),$$

(7) *being satisfied in the last two relations.*

Let us put $r = 0, 1$ in $D_1$, $s = n_j$ in $C_2$ and $r = 0$ in $D_2$ to prove this corollary.

The first two relations are generalizations of known results for exactly covering systems and $\varepsilon$-covering systems ([1], [2], [8]). In case $\mathfrak{m}(t) > 0$ for all $t$ (i.e. (3) covers the set of the all integers) we get from the first relation

$$\sum_{t=1}^{k}\frac{\mu_t}{n_t} \geqslant 1;$$

and the equation holds if and only if (3) is $(\mu, 1)$-covering system. In the opposite case, $n_0 > 1$ and $\mathfrak{m}(t) \geqslant 2$ for at least one $t$, so the inequality holds.

COROLLARY 2. *Let* $d > 1$ *be a positive integer with* $\delta_{1,d} = 0$. *Let* $n_{t_1} \leqslant n_{t_2} \leqslant \ldots \leqslant n_{t_r}$ *be all the moduli of the residue classes of* (3) *which are divisible by* $d$ (*if any*), *each of them appearing as many times as many times it appears in* (3). *Then*

1. *For every* $i = 1, \ldots, r$ *there exists a* $j$ *with* $j \neq i$, $j = 1, \ldots, r$ *and* $n_{t_i} | n_{t_j}$.

2. *The system* (3) *contains at least* $p$ *distinct residue classes modulo* $n_{t_r}$ *provided that* $p$ *is the least prime divisor of* $n_{t_r}$.

**Proof.** 1. The proof follows indirectly from $C_1$ for $s = 1$.

2. Let us put $j = t_r$, $s = 1$ in $C_1$. Then

$$\sum_{\substack{t=1\\ n_{t_r}|n_t}}^{k} \frac{\mu_t}{n_t} \exp\left(\frac{2\pi i}{n_{t_r}} a_t\right) = 0$$

because of $\delta_{1,n_{t_r}} = 0$. On the other hand $n_{t_r}$ is maximal and therefore $n_{t_r}|n_t$ only if $n_t = n_{t_r}$. Thus

$$\sum_{n_t=n_{t_r}} c_t \cdot \exp\left(\frac{2\pi i}{n_{t_r}} a_t\right) = 0,$$

where $c_t$ denotes the "appearance" of the residue class $a_t \pmod{n_t}$ in (3). The number $c_t$ could be also negative depending on $\mu$, but it is always non-zero due to our assumption in the introduction. From Theorem 1 in [4] we get that this sum contains at least $p$ addends where $p$ is the least prime divisor of $n_{t_r}$, and the proof of corollary is complete.

Evidently $\delta_{1,d} = 0$ for each $d > 1$ in case of $(\mu, 1)$-covering system. If $n_k$ is the greatest among the moduli $n_1, n_2, \ldots, n_k$ of the classes in (3), then for $d = n_k$ we get the known Znám's generalization of the result of Mirsky, Newmann, Davenport and Rado [1] from the second part of Corollary 2 (see also [5] and [8] for exactly covering systems and $\varepsilon$-covering systems). Since in general $n_{t_r}$ of Corollary 2 is not always the greatest modulus of classes in (3), Corollary 2 presents a generalization of Znám's result also for exactly covering systems.

Let us remark that the chain of moduli in Corollary 2 is empty for some $(\mu, \mathfrak{m})$-covering systems even for every $d > 1$. But the following rewritting of Corollary 2 shows when this is not the case.

COROLLARY 2′. *If the period $n_0$ of the system* (3) *is a proper divisor of* $[n_1, \ldots, n_k]$ *then*

1. *There exists at least one couple $n_i, n_j$ with $i \neq j$ of the moduli of* (3) *for which $n_i | n_j$.*

2. *The system* (3) *contains at least two distinct residue classes with respect to the same modulus.*

The first part of this corollary confirms the validity of the famous conjecture of Schinzel in case $n_0 < [n_1, \ldots, n_k]$ that in every covering system of congruences there is a couple of the moduli, one of which is a divisor of the other. This conjecture is also true if $n_0 = [n_1, \ldots, n_k]$ and $n_0$ is simultaneously one of the moduli of (3). Thus, there remains to solve the case $n_0 = [n_1, \ldots, n_k]$ and all the moduli less than $n_0$.

The condition of the second part of Corollary 2′ (that is the mentioned result of Mirsky, Newmann, Davenport and Rado if (3) is exactly covering)



is the best possible as the following example of [1] shows

$$0 \pmod 2, \quad 0 \pmod 3, \quad 1 \pmod 4, \quad 5 \pmod 6, \quad 7 \pmod{12},$$

but it is not necessary, e.g.

$$0 \pmod 2, \quad 0 \pmod 3, \quad 1 \pmod 6, \quad 5 \pmod 6.$$

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