

## On Hirzebruch sums and a theorem of Schinzel

by

P. CHOWLA and S. CHOWLA (Princeton, N. J.)

*Dedicated to Carl Ludwig Siegel on the occasion of his 75th birthday*

Let  $N$  be a positive square-free integer. Expressing  $\sqrt{N}$  as an infinite simple continued fraction we get

$$\sqrt{N} = a_0 + \underbrace{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_s}} + \dots,$$

where the "period" starts with the second term, and consists of  $s$  terms. We can also define  $s$  as the least positive integer such that  $a_s = 2a_0$ . Recently (the second author learnt this in oral conversation with Hirzebruch a few months ago) Hirzebruch proved the following astonishing and surprising theorem:

*If  $N$  is a prime  $\equiv 3(4) > 3$ , such that the class-number of  $\mathcal{Q}(\sqrt{+N})$  is 1, then*

$$\Sigma_N = a_s - a_{s-1} + a_{s-2} - a_{s-3} + \dots \pm a_1$$

*has the property*

$$\Sigma_N = 3h(-N).$$

*Here  $h(d)$  denotes the class-number of  $\mathcal{Q}(\sqrt{d})$ .*

This remarkable theorem led us to make the following conjectures.

Let  $N$  be a number  $\equiv 3(4)$ ,  $3 \nmid N$ . Then

- (i)  $\Sigma_N > 0$ ;
- (ii)  $3 \mid \Sigma_N$ .

Next let  $p$  be a prime  $\equiv 3(4)$ , then for  $p > 3$ ,

- (iii)  $\Sigma_p$  is an odd multiple of 3.



(iv) If  $N = 2p$ , where  $N$  is a prime  $\equiv 3(4)$  and  $h(2p) = 1$ , then

$$\Sigma_{2p} = 6h(-p).$$

Hence if  $p$  is a prime  $\equiv 3(4)$  and if  $h(p) = h(2p) = 1$  then ( $p = 4519$  is an example)

$$\Sigma_{2p} = 2\Sigma_p.$$

Both (ii) and (iii) have been proved by Andrzej Schinzel, in fact in generalized form. His beautiful proof is very elementary.

PENNSYLVANIA STATE UNIVERSITY  
INSTITUTE FOR ADVANCED STUDY

*Received on 26. 9. 1972*

(328)

Les volumes IV et suivants sont à obtenir chez	Volumes from IV on are available at	Die Bände IV und folgende sind zu beziehen durch	Томы IV и следу- ющие можно по- лучить через
--	---	--	--

**Ars Polona-Ruch, Krakowskie Przedmieście 7, 00-068 Warszawa**

Les volumes I-III sont à obtenir chez	Volumes I-III are available at	Die Bände I-III sind zu beziehen durch	Томы I-III можно получить через
--	-----------------------------------	---	------------------------------------

**Johnson Reprint Corporation, 111 Fifth Ave., New York, N. Y.**