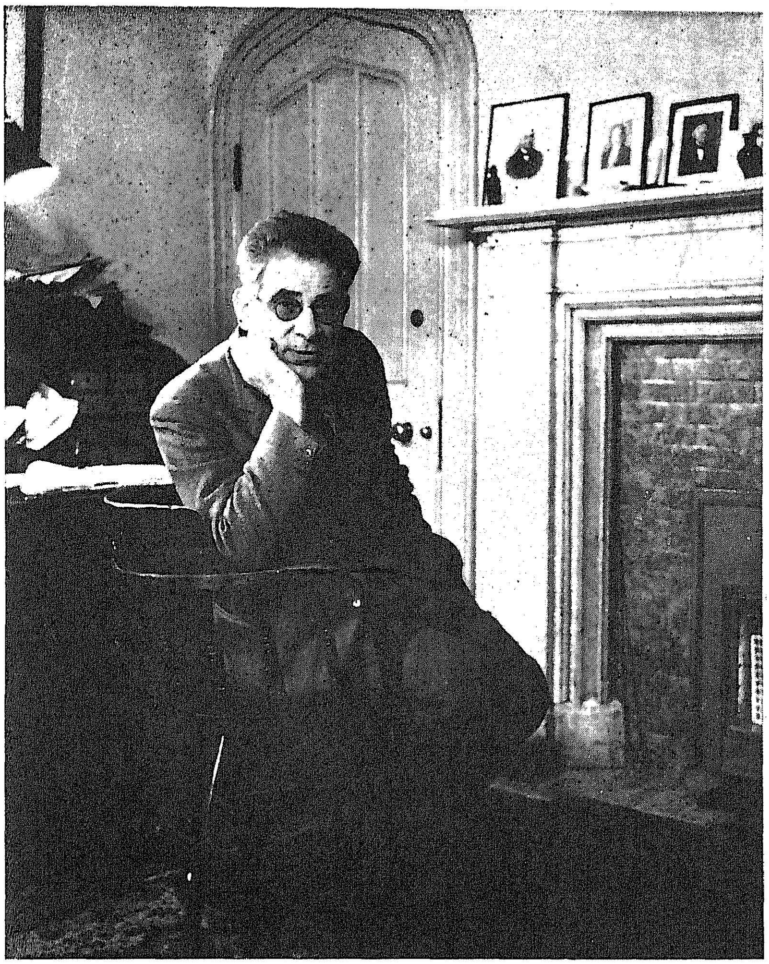



L. J. Mordell

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# Publications of L. J. Mordell

## Continuation from Acta Arithmetica, vol. IX, pp. 13-22

**Insert:**

133 (bis). *Review of Hasse's "Zahlentheorie"*, Math. Gazette 35 (1951), pp. 52–56.

133 (ter). *Review of Siegel's "Transcendental numbers"*, Math. Gazette 35 (1951), pp. 56–58.

**Complete the following references:**

203. *Incomplete exponential sums and incomplete residue systems for congruences*, Czechoslovakian J. of Math. 14 (89) (1964), pp. 235–242.

204. *On the least residue and non-residue of a polynomial*, J. London Math. Soc. 38 (1963), pp. 451–453.

205. *On a cubic exponential sum in three variables*, American J. Math. 85 (1963), pp. 49–52.

206. *On the congruence* $ax^3+by^3+cz^3+dxyz \equiv n \pmod{p}$, Duke Math. J. 31 (1964), pp. 123–126.

207. *The Diophantine equation* $y^2 = ax^3+bx^2+cx+d$ *fifty years after*, J. London Math. Soc. 38 (1963), pp. 454–458.

208. *The Diophantine equation* $y^2 = Dx^4+1$, J. London Math. Soc. 39 (1964), pp. 161–164.

**Add the following:**

209. *On the integer solutions of* $y(y+1) = x(x+1)(x+2)$, Pacific J. Math. 13 (1963), pp. 1347–1351.

210. *Review of Linnik and Gelfond. Elementary methods in analytic number theory*, Bull. Amer. Math. Soc. 70 (1964), pp. 653–658.

211. *Review of Serge Lang. Diophantine geometry*, Bull. Amer. Math. Soc. 70 (1964), pp. 491–498.

212. *On the conjecture for the rational points on a cubic surface*, J. London Math. Soc. 40 (1965), pp. 149–158.

213. *The Diophantine equation* $y^2 = ax^3+bx^2+cx+d$, Rend. Circ. Mat. Palermo 13 (2) (1964), pp. 1–8.

214. *The Diophantine equation* $y^2 = ax^3+bx^2+cx+d$, Scripta Math. 28 (1965), pp. 205–211.

215. *Review of Borevich and Shafarevich. Theory of numbers*, Bull. Amer. Math. Soc. 71 (1965), pp. 580–586.
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217. *Irrational Power Series II*, Acta Arith. 11 (1965), pp. 181–188.
218. *Review of Sierpiński. A selection of problems in the theory of numbers*, J. London Math. Soc. 40 (1965), pp. 571–573.
219. *Some quadratic Diophantine equations of genus 3*, Proc. Amer. Math. Soc. 17 (1966), pp. 1152–1158. *Addendum*, Ibid. 18 (1967), p. 198.
220. *The solvability of the equation $ax^2+by^2=p$*, J. London Math. Soc. 41 (1966), pp. 517–522.
221. *The infinity of rational solutions of $y^2=x^3+k$*, J. London Math. Soc. 41 (1966), pp. 523–525.
222. *Expansion of a function in terms of Bernouilli polynomials*, J. London Math. Soc. 41 (1966), pp. 526–528.
223. *Expansion of a function in a series of Bernouilli polynomials and some other polynomials*, J. Math. Anal. Appl. 15 (1966), pp. 132–140.
224. *Binary cubic forms expressed as a sum of cubes of seven linear forms*, J. London Math. Soc. 42 (1967), pp. 646–651.
225. *The Diophantine equation $z^2=ax^4+2bx^2y^2+cy^4$*, Math. Ann. 168 (1967), pp. 138–141.
226. *The representation of numbers by some quaternary quadratic forms*, Acta Arith. 12 (1966), pp. 47–54.
227. *Review of Chowla. The Riemann Hypothesis and Hilbert's 10th Problem*, J. London Math. Soc. 41 (1966), pp. 567–569.
228. *Some ternary quartic Diophantine equations*, Elem. Math. 21 (1966), pp. 89–90.
229. *Some exponential sums in several variables*, Monatsh. Math. 73 (1969), pp. 348–353.
230. *The Diophantine equation $x^4+my^4=z^2$*, Quart. J. Math. 18 (1967), pp. 1–6.
231. *The representation of a Gaussian integer as a sum of two squares*, Math. Mag. 40 (1967), p. 209.
232. *On numbers which can be expressed as a sum of powers*, Abhandlungen aus Zahlentheorie und Analysis zur Erinnerung an Edmund Landau. (VEB Deutscher Verlag der Wissenschaften, Berlin, 1968), pp. 219–221.
233. *On some Diophantine equations $y^2=x^3+k$ with no rational solutions (II)*, Ibid. pp. 225–232.
234. *The Diophantine equation $y^2=Dx^4+1$*, Colloquia Mathematica Societatis János Bolyai, 2 Number Theory, Debrecen (Hungary), (1968), pp. 141–145.



235. *Equations $ax^3+bx^2y+cxy^2+dy^3=c$ with no rational solution*, J. London Math. Soc. 43 (1968), pp. 433–438.
236. *The minimum value of a definite integral*, Math. Gazette 52 (1968), pp. 135–136.
237. *The diophantine equation $x^4+y^4=1$ in algebraic number fields*, Acta Arith. 14 (1968), pp. 347–355.
238. *The diophantine equations $y^2=x^4\pm 1$ in quadratic fields*, J. London Math. Soc. 44 (1969), pp. 112–114.
239. *The minimum value of a definite integral II*, Aequationes Math. 2 (1969), pp. 327–331.
240. *The Diophantine equation $dy^2=ax^4+bx^2+c$*, Acta Arith. 15 (1969), pp. 269–272.
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246. *On sums of four cubes of polynomials*, Acta Arith. 16 (1970), pp. 365–369.
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250. *Reminiscences of an octogenarian mathematician*, Amer. Math. Monthly 78 (1971), pp. 952–961.
251. *Harold Davenport (1907–1969)*, Acta Arith. 18 (1971), pp. 1–4.
252. *Some aspects of Davenport's work*, Acta Arith. 18 (1971), pp. 5–11.
253. *On sums of four cubes of polynomials* (with J. H. E. Cohn), J. London Math. Soc. 5 (1972), pp. 74–78.
254. *Review of C. G. Lekkerkerker "Geometry of numbers"*, Canad. Math. Bull. 14 (1971), pp. 611–613.
255. *On the representation of an integer as the sum of four integer cubes*, Computers in Number Theory (Ed. A. O. L. Atkin and B. J. Birch, Academic Press 1971), pp. 115–117.
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259. *On rational functions representing all residues* mod $p$, J. London Math. Soc. 5 (1972), pp. 166–168.
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261. *On the representation of positive integers as sums of three cubes of positive rational numbers*, Mathematika 18 (1971), pp. 98–99.
262. *A finite evaluation of a special exponential sum*, Proc. Camb. Phil. Soc. 71 (1972), pp. 75–78.
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265. *The sign of the Bernouilli numbers*, Amer. Math. Monthly (Math. Notes section), 80 (1973), pp. 547–548.
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267. *On the exponential sum* $\sum_{x=1}^{X} e\big(2\pi i(ax+bg^x)/p\big)$, Mathematika 19 (1972), pp. 84–87.
268. *A new type of exponential series*, Quart. J. Math. Oxford (2), 23 (1972), pp. 373–374.
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270. *Note on the product of n inhomogeneous linear forms*, J. Number Theory 4 (1972), pp. 405–407.

**Books**

*Diophantine Equations* (Academic Press, 1969), xii+312 pp.

The pamphlets *Three lectures on Fermat's last theorem* and *A chapter in the theory of numbers* (items 26 and 120) are being reprinted by the VEB Deutscher Verlag der Wissenschaften with an introduction by O. Neumann under the title: *Two papers on number theory*.

A Russian translation of *Reflections of a Mathematician* (item 167) has appeared under the title: *Размышления Математика* (Издательство Знание, Москва, 1971).

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# Waring's problem in quadratic number fields. Addendum

by

J. H. E. Cohn (London)

I am grateful to Professor P. T. Bateman for pointing out to me that there is some overlap between the results of [2] and those contained in [1] and [5]. In particular [2; Theorem 8] is a special case of [5; Theorem 10].

However, some of the results of [5] can be improved. Thus in the ring of Gaussian integers, it has been shown [3], that $g(3)\leqslant 4$, i.e. that every Gaussian integer is the sum of at most four cubes of Gaussian integers. It is easily seen that $g(3)\geqslant 3$ in this case, but which of the values 3 or 4 is the correct one is not known.

For fourth powers, we consider, again in the ring of Gaussian integers, two quantities $g(4)$ and $v(4)$, respectively the least number of fourth powers required to represent any member of $J_4$ as their sum, or as their sum or difference. In [4] it is shown that $g(4)\leqslant 18$, and in [5] that $g(4)\leqslant 14$ and $v(4)\leqslant 10$. We now show that $g(4)\leqslant 10$ and $v(4)\leqslant 8$. We have the identity

$$120x-131 = (2x+1)^4+(x-2+2i)^4+(x-2-2i)^4+\{(2+i)x\}^4+\\ +\{(2-i)x\}^4+\{(1+i)(x+1)\}^4,$$

and so if $\nu\equiv -11 \pmod{120}$, $\nu$ can be represented as the sum of six fourth powers. To conclude the proof that $g(4)\leqslant 10$, we observe that if $\nu\in J_4$ then $\nu\equiv 0$ or $\pm 1 \pmod 3$ and $\nu\equiv 0, \pm 1, \pm 2, \pm 3$ or $4 \pmod 8$ and it is easily seen that for any such $\nu$ it is possible to choose $\alpha, \beta_1, \beta_2, \beta_3, \beta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \delta_1, \delta_2, \delta_3$, and $\delta_4$ to satisfy

$$\nu-\alpha^4 \equiv 1 \pmod 3,$$
$$\nu-\beta_1^4-\beta_2^4-\beta_3^4-\beta_4^4 \equiv 5 \pmod 8,$$
$$\nu-\gamma_1^4-\gamma_2^4-\gamma_3^4-\gamma_4^4 \equiv -1 \pmod{1+2i},$$
$$\nu-\delta_1^4-\delta_2^4-\delta_3^4-\delta_4^4 \equiv -1 \pmod{1-2i}.$$