Publications of L. J. Mordell

Continuation from Acta Arithmetica, vol. IX, pp. 13-22

Insert:


Complete the following references:


Add the following:

214. The Diophantine equation $y^2 = ax^3 + bx^2 + cx + d$, Scripta Math. 28 (1965), pp. 205-211.
230. The Diophantine equation \( x^3 + mx^2 + nx^4 = 0 \), Quart. J. Math. 16 (1967), pp. 1–6.
232. On numbers which can be expressed as a sum of powers, Abhildungen aus Zahlentheorie und Analysis zur Erinnerung an Edmund Landau, (VEB Deutscher Verlag der Wissenschaften, Berlin, 1968), pp. 219–221.
234. The Diophantine equation \( y^2 = Dx^4 + 1 \), Colloquia Mathematica Societatis Janos Bolyai, 2 Number Theory, Debreccen (Hungary), (1968), pp. 141–145.
Waring's problem in quadratic number fields. Addendum
by J. H. E. Cohn (London)

I am grateful to Professor P. T. Bateman for pointing out to me that there is some overlap between the results of [2] and those contained in [1] and [5]. In particular [2; Theorem 6] is a special case of [3; Theorem 10].

However, some of the results of [3] can be improved. Thus in the ring of Gaussian integers, it has been shown [3], that $g(3) \leq 4$, i.e. that every Gaussian integer is the sum of at most four cubes of Gaussian integers. It is easily seen that $g(3) \geq 3$ in this case, but which of the values 3 or 4 is the correct one is not known.

For fourth powers, we consider, again in the ring of Gaussian integers, two quantities $g(4)$ and $v(4)$, respectively the least number of fourth powers required to represent any member of $J_4$ as their sum, or as their sum or difference. In [4] it is shown that $g(4) \leq 18$, and in [5] that $g(4) \leq 14$ and $v(4) \leq 10$. We now show that $g(4) \leq 10$ and $v(4) \leq 8$. We have the identity

$$120x - 131 = (2x + 1)^4 + (x - 2 + 2i)^4 + (x - 2 - 2i)^4 + (2 + i)^4 +$$

$$+ ((2 - i)x)^4 + (i + 1)(x + 1)^4,$$

and so if $x = -11 \pmod{120}$, $x$ can be represented as the sum of six fourth powers. To conclude the proof that $g(4) \leq 10$, we observe that if $x \in J_4$ then $x \equiv 0, \pm 1 \pmod{3}$ and $x = 0, \pm 1, \pm 2, \pm 3, 4 \pmod{8}$ and it is easily seen that for any such $x$ it is possible to choose $a, b, c, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \delta_1, \delta_2, \delta_3$ and $\delta_4$ to satisfy

$$v - a^4 \equiv 1 \pmod{3},$$

$$v - \beta_1^4 - \beta_2^4 - \beta_3^4 - \beta_4^4 \equiv 3 \pmod{8},$$

$$v - \gamma_1^4 - \gamma_2^4 - \gamma_3^4 - \gamma_4^4 \equiv -1 \pmod{12},$$

$$v - \delta_1^4 - \delta_2^4 - \delta_3^4 - \delta_4^4 \equiv -1 \pmod{24}.$$