It is easy to see in (ii) and (iii) that A satisfies (5) and $G(A) \geqslant 2n+4K+1$.

The examples in (i), (ii) and (iii) together with (6) establish the theorem for $k = K$. This completes the induction step and the theorem is proved.

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Added in proof: The conjecture $g(3, t) = \left[\frac{(t-2)^2}{2}\right] - 1$ has recently been settled in the affirmative by M. Lewin (personal communication).

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Remarks on some new applications of the dispersion method

by

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Dispersion method as expounded in the works [1] and [2] can be applied to proving a general result on the equation

$$n = \frac{\nu_1\varphi_1 - \nu_2\varphi_2}{\nu_1 - \nu_2}$$

for large n's; ν_i, φ_i being rather general system of numbers the equation is solvable, and a *lower estimate of the asymptotic* can be obtained. The particular cases are:

The equation:

$$n = \frac{p_1 p - p_1' p'}{p_1 - p_1'} \qquad \text{(A)}$$

with p, p', p_1, p_1' primes, $p \leqslant n, p_1, p_1' \leqslant (\ln n)^a$; $a > e$ has the number of solutions:

$$Q_A(n) \geqslant (\ln a)(\ln a - 1)\frac{n}{\ln n} + O\left(\frac{n}{\ln n \ln\ln n}\right).$$

The equation:

$$2 = \frac{p_1 p - p_1' p'}{p_1 - p_1'} \qquad \text{(B)}$$

with p, p', p_1, p_1' as above, $n \to \infty$ has the number of solutions:

$$Q_B(n) \geqslant \ln a(\ln a - 1)\frac{n}{\ln n} + O\left(\frac{n}{\ln n \ln\ln n}\right).$$

The equation:

$$n = \frac{p_1^r p - p_1'^r p'}{p_1^r - p_1'^r} \qquad \text{(C)}$$

with p, p' primes $\leqslant n$; $(\ln n)^{c_1} \leqslant p_1^r, p_1'^r \leqslant (\ln n)^{c_2}$, $r = 2, 3, 4, \ldots$ a given constant; $c_2 - \frac{c_2}{r} + 1 < c_1$ has the number of solutions:

$$Q_G(n) \geqslant \frac{n}{(\ln n)^{2c_2 - \frac{2c_2}{r} + r - c_1}} \big(1 + o(1)\big).$$

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A remark on a previous paper by Bredihin and Linnik

by

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In memory of W. Sierpiński

In this note we give a short and simple proof of the following theorem which was a basis for Bredihin's and Linnik's considerations concerning certain diophantine equations [1]. Our argument is in fact just a simple modification of that used by those authors.

Theorem A. *Let n be a given complex number, let ν and φ range independently without repetitions over finite systems of complex values from given domains (ν) resp. (φ) and let D be an arbitrary complex number. We denote by $S(n)$ the number of solutions in ν, φ, D of the equation*

$$\nu D + \varphi = n \tag{1}$$

where $\nu \in (\nu)$ and $\varphi \in (\varphi)$. Further, we define

$$T(n) = \sum_{\substack{D \\ S(n,D) \neq 0}} 1$$

where $S(n, D)$ is the number of solutions in $\nu \in (\nu)$, $\varphi \in (\varphi)$ of (1) for a fixed D, and finally we put

$$\mathrm{Def}(n) = S(n) - T(n).$$

Then the number $Q(n)$ of all possible representations of n in the form

$$n = \frac{\nu_1 \varphi_1 - \nu_2 \varphi_2}{\nu_1 - \nu_2} \quad (\nu_1 \neq \nu_2)$$

with $\nu_1, \nu_2 \in (\nu)$, $\varphi_1, \varphi_2 \in (\varphi)$ satisfies the estimation

$$Q(n) \geqslant \frac{S(n)}{T(n)} \mathrm{Def}(n). \tag{2}$$

Proof. For any non-negative integer r consider the number N_r of all those complex numbers D for which equation (1) has exactly r solutions in ν, φ, $\nu \in (\nu)$, $\varphi \in (\varphi)$. Clearly

$$S(n) = \sum_{r \geqslant 0} r N_r = \sum_{r \geqslant 1} r N_r$$