with p, p' primes $\leqslant n$; $(\ln n)^{c_1} \leqslant p_1^r, p_1'^r \leqslant (\ln n)^{c_2}$, $r = 2, 3, 4, \ldots$ a given constant; $c_2 - \frac{c_2}{r} + 1 < c_1$ has the number of solutions:

$$Q_C(n) \geqslant \frac{n}{(\ln n)^{2c_2 - \frac{2c_2}{r} + r - c_1}} \big(1 + o(1)\big).$$

References

[1] B. M. Bredihin, *Dispersion method and definite binary problems* (Russian), Uspehi Mat. Nauk, 20 (2) (1965), pp. 89–130.

[2] Yu. V. Linnik, *Dispersion method in binary additive problems*, Amer. Math. Soc. edition, 1963.

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A remark on a previous paper by Bredihin and Linnik

by

S. Uchiyama (Okayama)

In memory of W. Sierpiński

In this note we give a short and simple proof of the following theorem which was a basis for Bredihin's and Linnik's considerations concerning certain diophantine equations [1]. Our argument is in fact just a simple modification of that used by those authors.

Theorem A. *Let n be a given complex number, let ν and φ range independently without repetitions over finite systems of complex values from given domains (ν) resp. (φ) and let D be an arbitrary complex number. We denote by $S(n)$ the number of solutions in ν, φ, D of the equation*

$$\nu D + \varphi = n \tag{1}$$

where $\nu \in (\nu)$ and $\varphi \in (\varphi)$. Further, we define

$$T(n) = \sum_{\substack{D \\ S(n,D) \neq 0}} 1$$

where $S(n, D)$ is the number of solutions in $\nu \in (\nu)$, $\varphi \in (\varphi)$ of (1) for a fixed D, and finally we put

$$\mathrm{Def}(n) = S(n) - T(n).$$

Then the number $Q(n)$ of all possible representations of n in the form

$$n = \frac{\nu_1\varphi_1 - \nu_2\varphi_2}{\nu_1 - \nu_2} \quad (\nu_1 \neq \nu_2)$$

with $\nu_1, \nu_2 \in (\nu)$, $\varphi_1, \varphi_2 \in (\varphi)$ satisfies the estimation

$$Q(n) \geqslant \frac{S(n)}{T(n)} \mathrm{Def}(n). \tag{2}$$

Proof. For any non-negative integer r consider the number N_r of all those complex numbers D for which equation (1) has exactly r solutions in ν, φ, $\nu \in (\nu)$, $\varphi \in (\varphi)$. Clearly

$$S(n) = \sum_{r \geqslant 0} rN_r = \sum_{r \geqslant 1} rN_r$$

and

$$T(n) = \sum_{r\geq 1}{}' N_r .$$

Applying the Cauchy-Schwarz inequality we get

$$\Big(\sum_{r\geq 1}{}' rN_r\Big)^2 \leq \sum_{r\geq 1}{}' r^2 N_r \cdot \sum_{r\geq 1}{}' N_r,$$

so

$$\sum_{r\geq 1}{}' r^2 N_r \geq \frac{(S(n))^2}{T(n)}.$$

Since

$$Q(n) \geq \sum_r{}' r(r-1)N_r = \sum_r{}' r^2 N_r - \sum_r{}' rN_r,$$

inequality (2) follows and the theorem is proved.

We remark that our argument may also be applied to some "dual" problems dealt with in the last section of the paper by Bredihin and Linnik [2].

References

[1] Б. М. Бредихин, Ю. В. Линник, *Некоторые диофантовые уравнения с простыми числами*, Математические Заметки 12 (3) (1972).

[2] B. M. Bredihin and Yu. V. Linnik, *Remarks on some new applications of the dispersion method*, Acta Arith., this volume, pp. 409–410.

OKAYAMA UNIVERSITY
Okayama, Japan

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Additive problems involving squares, cubes and almost primes

by

Yu. V. Linnik

We consider here certain additive problems of the binary type in the sense of [3] § 1, namely the representation of large numbers by the sums of two squares and two cubes, two squares and three cubes, one square and a ternary cubic form and a product of two primes and a ternary cubic form. Only in the case of two squares and three cubes we succed in solving the corresponding equation for all large numbers; in other cases we solve the equations for certain large classes of numbers. No asymptotic is obtained, only certain crude lower estimates are obtained. However in same cases the possibility of obtaining asymptotic formulae with help of the dispersion method [3] is indicated.

§ 1. Consider the diophantine equations:

$$n = \xi^2+\eta^2+x^3+y^3, \tag{1.1}$$

$$n = \xi^2+\eta^2+x^3+y^3+z^3 \tag{1.2}$$

with non-negative ξ, η, x, y, z.

There are many reasons to believe that the equation (1.1) is solvable for all large numbers n, but we cannot prove it. We shall consider here only even numbers $n = 2N_1$. We can represent any even number n, in the form $n = 2^{3\mu}3^{6\nu}2^{\alpha}3^{\beta}n_1$ where $1 \leq \alpha \leq 3$; $0 \leq \beta \leq 5$; $\mu \geq 0, \nu \geq 0$, $(n_1, 6) = 1$. We shall call the corresponding number $2^{\alpha}3^{\beta}n_1$ the kernel of even number n. Clearly if $2^{\alpha}3^{\beta}n_1$ is representable in the form (1.1) so is n. Therefore we shall consider only the kernels of the even numbers n.

In what follows, $c_0, c_1, c_2, \ldots, K_0, K_1, K_2, \ldots$ will be positive constants, $\varepsilon_0, \varepsilon_1, \ldots$ small positive constants.

THEOREM 1. *Let $\Gamma(K_0, K_1, K_2)$ be the set of all even numbers n subject to conditions*: 1) *the kernels* $2^{\alpha}3^{\beta}n_1 \geq K_0$; 2) *the number* n_1 *has*