leisurely rates. The five functions of the following table refer to partitions into distinct parts, parts \( > 1 \), unrestricted parts, odd parts and even parts respectively.

### Table II

<table>
<thead>
<tr>
<th>( n )</th>
<th>( W_n^* )</th>
<th>( W_n(S_n) )</th>
<th>( W_n/S_n )</th>
<th>( W_n(S_n)/\sqrt{\pi n} )</th>
<th>( \sqrt{n}W_n(S_n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>.566786</td>
<td>.556790</td>
<td>.542158</td>
<td>.600193</td>
<td>1.072995</td>
</tr>
<tr>
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<td>.566736</td>
<td>.555423</td>
<td>.642586</td>
<td>.600202</td>
<td>1.071101</td>
</tr>
<tr>
<td>102</td>
<td>.566691</td>
<td>.555589</td>
<td>.642625</td>
<td>.609277</td>
<td>1.071101</td>
</tr>
<tr>
<td>103</td>
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<td>.555546</td>
<td>.642560</td>
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<td>1.072575</td>
</tr>
<tr>
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<td>.555011</td>
<td>.642680</td>
<td>.608432</td>
<td>1.072575</td>
</tr>
<tr>
<td>Limit</td>
<td>.561450</td>
<td>.561450</td>
<td>.561450</td>
<td>.561459</td>
<td>.674612</td>
</tr>
</tbody>
</table>

The slight irregularities in these functions are not due to inaccuracy. They reflect the existence of an asymptotic, or possibly convergent, series for each entry.

### Reference


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**Some diophantine equations solvable by identities**

by

A. Mąkowski (Warszawa)

### Dedicated to the memory of my teacher Wacław Sierpiński

1. W. Sierpiński in many of his papers investigated the triangular numbers \( t_n = \frac{1}{2}n(n+1) \) and tetrahedral numbers \( T_n = \frac{1}{4}n(n+1)(n+2) \). From the identity given by A. Gérardin [1] we get immediately the following identity

\[
(27n^6)^2 - 1 = (9n^4 - 3n^2 + (9n^3 - 1)^3 = (9n^4 + 3n^2)^3 - (9n^3 + 1)^3.
\]

With \( n \) odd and positive the last identity provides infinitely many integer solutions of the equation

\[
(2x+1)^2 - 1 = (2y)^3 + (2z)^3 = (2u)^3 - (2v)^3
\]

which is equivalent to

\[
t_n = y^3 + z^3 = u^3 - v^3.
\]

Thus there exist infinitely many triangular numbers which are simultaneously representable as sums and differences of two positive cubes.

We have the identity \( 3aT_{n-1} = t_{3n-1} \). Since there exist infinitely many tetrahedral numbers divisible by 3: \( T_n = 3a \) we infer that there exist infinitely many triangular numbers which are products of two tetrahedral numbers \( > 1 \).

2. The numbers \( x = 6^3p^2n^3 + 6^3p^4r^4n^4, \ y = 6^3p^2r^2n^2 - 6^3p^4r^4n^4, \ z = 6^3p^2r^2n^4 \) satisfy the equation

\[
p(x^3 + y^3 - z^3) = r(x - y).
\]

This answers a question posed by A. Oppenheim in [3].

3. L. J. Mordell [2] investigated the equation \( z^2 = ax^3 + by^3 + c \). It may be noticed that the equation

\[
z^2 = ax^{2k+1} + by^{2k+1} + c
\]
The representation of real numbers by infinite series of rationals

by

A. Oppenheim (Legon, Ghana)

Received on 19. 8. 1971

1. In a recent note Galambos [1] has obtained some remarkable theorems about the ergodic properties of the denominators in the expansion

\[ x = \frac{1}{a_1} + \frac{1}{b_1} + \frac{1}{a_2} + \frac{1}{b_1 b_2} + \frac{1}{a_3} + \frac{1}{b_1 b_2 b_3} + \cdots; \]

he refers (Ref. 9 in Galambos [1]) to unpublished work of mine on this expansion. It seems appropriate now to give detailed results.

The expansion for any \( x > 0 \) (not necessarily confined to the interval \((0, 1)\)) derives from the algorithm

\[ a = x_1, \quad d_i = 1 + [1/x_i], \quad a_i = 1/d_i + (a_i/b_i)x_{i+1}; \]

for \( i = 1, 2, \ldots \) Herein

\[ a_i = a_i(d_1, d_2, \ldots, d_i), \quad b_i = b_i(d_1, d_2, \ldots, d_i) \]

are positive numbers (usually integers).

Several questions arise:

(i) to give conditions to ensure that the infinite series (necessarily convergent) in (1.1) has sum \( s \);

(ii) to obtain the conditions induced by the algorithm on the integers \( d_i \geq 1 \) (one such condition is

\[ d_{i+1} > (a_i/b_i)d_i(d_i - 1); \]

(iii) to obtain necessary and sufficient conditions that a convergent infinite series (1.1) shall be the expansion of its sum by the algorithm.

(A simple set of sufficient conditions is given by

\[ d_{i+1} - 1 \geq (a_i/b_i)d_i(d_i - 1). \]