leisurely rates. The five functions of the following table refer to partitions into distinct parts, parts >1, unrestricted parts, odd parts and even parts respectively.

Table II

n	W_n^*	$W_n(S_1)$	W_n/n	$W_n(S_w)/\sqrt{n}$	$\sqrt{n}W_n(S_e)$
100	.566786	.555790	.542158	.669193	1.072995
101	.566726	.555423	.542289	.669292	0
102	.566691	.555910	.542423	.669277	1.071161
103	.566634	.555546	.542550	.669369	0
104	.566584	.556011	.542680	.669432	1.072575
Limit	.561459	.561459	.561459	.674612	1.05968

The slight irregularities in these functions are not due to inaccuracy. They reflect the existence of an asymptotic, or possibly convergent, series for each entry.

Reference

[1] G. H. Hardy, *Divergent Series*, Oxford 1949.

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Some diophantine equations solvable by identities

by

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Dedicated to the memory of my teacher Wacław Sierpiński

1. W. Sierpiński in many of his papers investigated the triangular numbers $t_n = \frac{1}{2}n(n+1)$ and tetrahedral numbers $T_n = \frac{1}{6}n(n+1)(n+2)$.

From the identity given by A. Gérardin [1] we get immediately the following identity

$$(27n^6)^2-1 = (9n^4-3n)^3+(9n^3-1)^3 = (9n^4+3n)^3-(9n^3+1)^3.$$

With n odd and positive the last identity provides infinitely many integer solutions of the equation

$$(2x+1)^2-1 = (2y)^3+(2z)^3 = (2u)^3-(2v)^3$$

which is equivalent to

$$t_x = y^3+z^3 = u^3-v^3.$$

Thus there exist infinitely many triangular numbers which are simultaneously representable as sums and differences of two positive cubes.

We have the identity $3aT_{a-1} = t_{a^2-1}$. Since there exist infinitely many tetrahedral numbers divisible by 3: $T_m = 3a$ we infer that there exist infinitely many triangular numbers which are products of two tetrahedral numbers >1.

2. The numbers $x = 6^2pr^2n^3+6^6p^4r^5n^9$, $y = 6^2pr^2n^3-6^6p^4r^5n^9$, $z = 6^5p^3r^4n^7$ satisfy the equation

$$p(x^3+y^3-z^3) = r(x-y).$$

This answers a question posed by A. Oppenheim in [3].

3. L. J. Mordell [2] investigated the equation $z^2 = ax^3+by^3+c$. It may be noticed that the equation

$$z^2 = ax^{2k+1}+by^{2k+1}+c$$

($|a|+|b|>0$) may have at least n integer solutions (n given) for $c=2(ab)^{k+1}d^{2k+1}$ and appropriately chosen d, viz. $x=ap^2$, $y=bq^2$, $z=a^{k+1}p^{2k+1}+b^{k+1}q^{2k+1}$, where p, q are complementary divisors of d, i.e. $d=pq$.

4. The equation

$$(x+k)\ldots(x+kn)=y(y+1)\ldots(y+n)$$

has a solution for every k and n: $x=k^{n+1}$, $y=k^n$.

References

[1] A. Gérardin, L'intermédiaire des math. 19 (1912), p. 7.

[2] L. J. Mordell, *Note on the integer solution of* $z^2-k^2=ax^3+by^3$, Ganita 5 (1954), pp. 103–104.

[3] A. Oppenheim, *On the diophantine equation* $x^3+y^3-z^3=px+py-qz$, Univ. Beograd. Publ. Elekrotehn. Fak. Ser. Mat. Fiz. no. 235 (1968).

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ACTA ARITHMETICA
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The representation of real numbers by infinite series of rationals

by

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Wacław Sierpiński in memoriam

1. In a recent note Galambos [1] has obtained some remarkable theorems about the ergodic properties of the denominators in the expansion

$$x\sim\frac{1}{d_1}+\frac{a_1}{b_1}\cdot\frac{1}{d_2}+\frac{a_1a_2}{b_1b_2}\cdot\frac{1}{d_3}+\ldots; \tag{1.1}$$

he refers (Ref. 9 in Galambos [1]) to unpublished work of mine on this expansion. It seems appropriate now to give detailed results.

The expansion for any $x>0$ (not necessarily confined to the interval $(0,1)$) derives from the algorithm

$$x=x_1,\quad d_i=1+[1/x_i],\quad x_i=1/d_i+(a_i/b_i)x_{i+1} \tag{1.2}$$

for $i=1,2,\ldots$ Herein

$$a_i=a_i(d_1,d_2,\ldots,d_i),\quad b_i=b_i(d_1,d_2,\ldots,d_i)$$

are positive numbers (usually integers).

Several questions arise:

(i) to give conditions to ensure that the infinite series (necessarily convergent) in (1.1) has sum x;

(ii) to obtain the conditions induced by the algorithm on the integers $d_i\geqslant 1$ (one such condition is

$$d_{i+1}>(a_i/b_i)d_i(d_i-1)); \tag{1.3}$$

(iii) to obtain necessary and sufficient conditions that a convergent infinite series (1.1) shall be the expansion of its sum by the algorithm. (A simple set of sufficient conditions is given by

$$d_{i+1}-1\geqslant(a_i/b_i)d_i(d_i-1).) \tag{1.4}$$