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Corrigendum to the paper "On the probability that n and $f(n)$ are relatively prime"

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by

R. R. HALL (York)

The proof of Lemma 2 is incorrect, for it assumes that $p \leqslant x/\tilde{\omega}$ for every $\tilde{\omega}$ in the sum, whereas we only know that $p \leqslant x$. We therefore have to find an alternative treatment relying on Lemma 1 only. Lemma 3 becomes:

LEMMA 3′. *For $p \leqslant \sqrt{x}$ and all a,*

$$\sum_{\substack{m \leqslant x \\ g(m) \equiv a \pmod p}} 1 \leqslant C_{10} x \left(\frac{1}{\sqrt{p}} + \sqrt{\frac{\log p}{\log x}}\right).$$

From this we deduce

LEMMA 5′. *For all $\xi > 0$,*

$$\sum_{\substack{n \leqslant x \\ \exists p \mid (n, g(n)) \\ p > \xi}} 1 \leqslant \frac{C_{14} x}{\xi^{1/2}} + O\left(x \sqrt{\frac{\log\log x}{\log x}}\right).$$

Proof. The sum on the left does not exceed

$$\sum_{\xi < p < H} \sum_{\substack{m \leqslant x/p \\ g(m) \equiv 0 \pmod p}} 1 + \sum_{m \leqslant x/H} \sum_{\substack{p \mid g(m) \\ p > H}} 1.$$

Provided $H \leqslant x^{1/3}$, so that in the first sum, $p \leqslant \sqrt{x/p}$, we have, by Lemma 3′,

$$\sum_{\substack{n \leqslant x \\ \exists p \mid (n, g(n)) \\ p > \xi}} 1 \leqslant C_{10} \sum_{\xi < p \leqslant H} \frac{x}{p} \left(\frac{1}{\sqrt{p}} + \sqrt{\frac{\log p}{\log (x/p)}}\right) + \pi(x) + \sum_{2 \leqslant m \leqslant x/H} \frac{2 \log m}{\log H}$$

$$\leqslant \frac{C_{14} x}{\xi^{1/2}} + O\left(x \sqrt{\frac{\log H}{\log x}} + \frac{x \log x}{H \log H}\right),$$

using the upper bound $(2\log m)/\log H$ for the number of distinct prime factors exceeding H of $g(m)$, since $2 \leqslant g(m) \leqslant m^2$ for $m \geqslant 2$. Choosing $H = \log^2 x$ we obtain our result, and this leads to the slightly better

$$T(x) = \frac{6}{\pi^2} x + O\left(\frac{x}{\sqrt{\log\log\log x}}\right)$$

the rest of the proof being unchanged.