H Davenport

HAROLDO DAVENPORT
in memoriam

ACTA ARITHMETICA
XVIII (1971)

Harold Davenport (1907–1969)

by

L. J. MORDELL (Cambridge)

Professor Harold Davenport was born on October 30th 1907 near Accrington, Lancashire, England, and died on June 9th 1969 in Cambridge, England. He attended Accrington Grammar School where his interest in mathematics was aroused by his teacher, a Miss Heap, of whom he spoke very highly and to whom he was very grateful. He then studied at Manchester University from 1924–27 and then at Trinity College, Cambridge. He distinguished himself in the examinations at both Manchester and Cambridge. He was awarded a Fellowship at Trinity College in 1932 and stayed there until 1937. He then joined the staff at Manchester University as an assistant lecturer. In 1941, he was appointed Professor at the University College of North Wales at Bangor and then in 1945 to the Astor Professorship of Pure Mathematics in the University of London, tenable at University College. Finally, in 1958, he was elected to the Rouse Ball Professorship at Cambridge.

He was visiting Professor for various periods at a number of Universities both in the U. S. A. and on the Continent, for example, at Stanford, Ann Arbor, and Milan; and was Gauss Professor at the Akademie der Wissenschaften in Göttingen for the Sommersemester of 1966. He also gave individual lectures in many institutions and at conferences, in a great many places.

The originality and excellence of his work were universally recognised from the very beginning. He was elected a Fellow of the Royal Society of London at an early age in 1940 while he was still an assistant lecturer, a most unusual occurrence for such a junior person. An essay *On the geometry of numbers — on Waring's Problem* was awarded the Adams Prize of the University of Cambridge in 1941 and he received the Sylvester Medal of the Royal Society in 1967. He received the degree of Sc. D. at Cambridge in 1938 and the honorary degree of D. Sc. from Nottingham University in 1968. In 1964, he was elected an ordinary member of the Royal Society of Sciences at Uppsala. He was President of the London Mathematical Society 1957–59 and received its senior Berwick Prize in 1954. He founded a new mathematical journal, Mathematika, sponsored by University College, London. His editorial services were of the greatest value.

He has published more than 190 papers and has written the following books:

The Higher Arithmetic, Hutchinson's University Library, 3rd edition, 1968.

Multiplicative Number Theory, Markham, Chicago, 1967.

Analytic methods for Diophantine equations and Diophantine inequalities, Ann Arbor Publishers, Michigan, 1962 (Lectures at the University of Michigan, Fall Semester 1962).

I came across Davenport as a first year student at Manchester. During his undergraduate career, he gave every sign of being a real mathematician with a promising career before him and the future more than justified this. I was very much impressed by the elegance of his solutions of various problems, the logical precision of his style and the beauty of his handwriting. These characteristics he displayed throughout his life and his manuscripts were beautifully written.

His interest in number theory seems to be due to Littlewood, who proposed to him a problem which Davenport dealt with in his first paper.

In the early thirties, he spent much time in Marburg with Hasse to whom he taught English and from whom he learnt German.

When Davenport returned from Trinity College to Manchester, he could not have come to a better place or at a better time, for this was the beginning of the Manchester school of number theory. We had there K. Mahler, now F. R. S., later Professor at Manchester, Canberra and Ohio State at Columbus, Paul Erdös, Professor of the Academy of Sciences in Budapest, H. Heilbronn, now F. R. S., Professor at Bristol and Toronto; B. Segre, now President of the Lincei Academy at Rome. There were also Chao Ko, now Professor in China who has recently done some brilliant work on Diophantine Equations; and others. There were many visitors among whom might be mentioned Professor Chabauty, Professor D. H. Lehmer, Dr., now Professor, Rankin. It was not surprising that number theory flourished at Manchester. Really important work was done, inter alia, in the geometry of numbers, diophantine equations and congruences. We all benefited enormously from the association with each other, and I in particular from a study of Davenport's work.

The existence and success of a mathematical school obviously depend on the personality of its head, and in particular upon the interest he takes in his students, by the encouragement he gives them and frequently by his help in suggesting topics for research. Davenport had all the

qualities required for being a stimulating leader. He had not been long in London when he developed a very flourishing research school, and he did this also in Cambridge. Many of the younger brilliant mathematicians came under his influence. Among these may be mentioned C. A. Rogers, K. F. Roth, the first British Fields' Medallist, D. A. Burgess, A. Baker (1). He was a painstaking director of research and many lesser lights are greatly indebted to him not only for ideas but also for the care and patience he gave them, and his help in writing and producing their theses. Under his leadership, the Cambridge number theory school must have been one of the most important and flourishing schools to be found anywhere in the world.

This accounts for the large number of visitors from outside Britain, who attended his seminar. Many of them found great stimulation in working with him and a list of those who have collaborated with him in joint papers is most impressive. There were about seventy-five joint papers and twenty-four joint authors. These included practically all those who came in contact with him. These were, in the order in which the joint work with him appeared, Hasse, Heilbronn, Erdös, Mahler, van der Corput, C. A. Rogers, Pólya, Marshall Hall Jr., Chatland, Bambah, G. L. Watson, Swinnerton-Dyer, K. F. Roth, Birch, Ridout, S. Chowla, D. J. Lewis, Schinzel, LeVeque, Bombieri, Halberstam, W. M. Schmidt, Landau, A. Baker. Though I had been closely associated with him for many years, no joint paper arose because I can work only in my study. We had however a great many interests in common and some of my work attracted his attention. I am very grateful to him for his helpful criticism from which I benefited greatly. He often improved my results and some times found simpler proofs. Whatever I could do, he could usually do better.

Perhaps one can summarize Davenport's work by saying that it was characterized by originality, beauty and power, qualities which are not often found in the same person. Moreover he could deal systematically and accurately with the formidable arithmetical details required in some of his work. He had all the mathematical virtues one would like to have. Whatever he did, he did exceedingly well, probably because he had a very logical mind and so made straight for his goal. Consequently none of his work reflected either his great love of mathematics or the excitement he found in doing it.

He was one of the best and clearest lecturers I have ever heard and has a most attractive style. He was in great demand as a lecturer.

(1) Also a Fields' Medallist, 1970.

Fortunately many of his lectures were either published or mimeographed and will be a source of pleasure for many years to come.

His early death is a great loss to mathematics and to all who knew him. I count myself very fortunate to have known him for some 45 years. He is assured of a permanent place among those great mathematicians who have advanced the theory of numbers, called by Gauss, the Queen of Mathematics.

ACTA ARITHMETICA
XVIII (1971)

Some aspects of Davenport's work

by

L. J. MORDELL (Cambridge)

It is well known what an important part has been played by problems, even of the simplest character, in furthering research, discovery and the advancement of mathematics. Hilbert's famous address on problems is a classic illustration. The solution of a problem frequently requires new ideas and new methods. The generalization it suggests, its consideration from a different point of view or its rephrasing may lead to a new problem of far greater significance than the original one which may turn out to be only a very special case of a general theorem. Sometimes it seems almost incredible what striking and far-reaching fundamental developments have arisen in directions which seem very remote indeed from the problem from which they arose.

Problems are the life-blood of mathematics. Davenport wrote nearly two hundred papers and in them he studied a great variety of problems on number theory and related topics. These stimulated a great deal of research and often proved the starting point for much work by his colleagues and students. Two problems in particular led him and others to investigations of the greatest importance which have greatly enriched mathematics, opening up entirely new and unexpected fields of research. Even at the present time, their possibilities have not been exhausted.

The first problem was tackled in his very first paper published in 1931. It is really wonderful what this led to and it makes a fascinating story to follow its consequences and to discuss the influence it had in shaping some of the best mathematical research for many years. I propose to do this in some detail. The problem, proposed to him by Littlewood, was to estimate the sum

$$S = \sum_{x=0}^{p-1}\left(\frac{(x+a)(x+b)(x+c)}{p}\right) = \sum_{x=0}^{p-1}\left(\frac{f(x)}{p}\right), \tag{1}$$

say, where p is a large prime and the bracket denotes the Legendre quadratic character mod p. The case when $f(x)$ is a quadratic polynomial is really very simple and had been investigated by Jacobsthal as long