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On the divisibility of $\sigma_\nu(n)$*

by

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I. Introduction

The divisor function $\sigma_\nu(n)$ is defined by

$$\sigma_\nu(n) = \sum_{d|n} d^\nu$$

where the sum is over all positive integral divisors of n; in the following pages it will be assumed that ν is a positive integer. The aim of this paper is to investigate a certain divisibility property of $\sigma_\nu(n)$.

Let q be a prime and m a positive integer, and assume that both are fixed and independent of x. Denote by $D_m(\nu, q; x)$ the number of positive integers $n \leqslant x$ for which $q^m \| \sigma_\nu(n)$, where the notation $\|$ means that q^m divides $\sigma_\nu(n)$ but q^{m+1} does not. In this paper an asymptotic equation for $D_m(\nu, q; x)$ will be established. Define γ by $q^\gamma \| \nu$, and let $m' = [m/(\gamma+1)]$ and $h = (q-1)/(\nu, q-1)$. Then the precise result to be obtained is as follows:

Theorem 1. (i) *If q and h are both odd, then, as $x \to \infty$,*

$$D_m(\nu, q; x) \sim A_1^{(m)} x.$$

(ii) *If q is odd and h is even, then, as $x \to \infty$,*

$$D_m(\nu, q; x) \sim A_2^{(m)} x (\log\log x)^{m'} (\log x)^{-1/h}.$$

(iii) *As $x \to \infty$,*

$$D_m(\nu, 2; x) \sim A_3^{(m)} x (\log\log x)^{m-1} (\log x)^{-1}.$$

$A_1^{(m)}$, $A_2^{(m)}$, $A_3^{(m)}$ are positive constants depending only on ν, q and m.

The corresponding results for the case $m = 0$ have been obtained by R. A. Rankin in a paper [1] published in 1961. The function $D_0(\nu, q; x)$

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